Math 4740 - Homework # 4
Random Variables, Expected Value, Games

Part 1 - The probability function and expected value of X

---

1. (a) Suppose you roll an 6-sided die. If you roll a 1 or 5 then you win \$1. If you roll a 2 or 4 or 6 then you win \$5. If you roll a 3 then you lose \$15. Let X be the amount of money won or lost on this game. Do the following: (i) Draw a picture of X, (ii) draw a picture of the probability function p, and (iii) calculate the expected value $E[X]$.

   (b) Suppose you have the same payouts as in part (a) of this problem, but instead you have a weighted non-fair 6-sided die. You know that the probabilities of each number are: 1 and 5 have probability 1/6 each, 2 and 4 and 6 have probabilities 1/12 each, and 3 has probability 5/12. What is the expected value of this game?

2. Suppose there are two coins. If you flip coin A it will land on heads with probability 0.6 and tails with probability 0.4. If you flip coin B it will land on heads with probability 0.7 and tails with probability 0.3.

   Assume that the two coins outcomes are independent of each other. The experiment that you perform is you first flip coin A and then you flip coin B.

   Let X denote the number of heads the occur when you flip the two coins. Thus X can equal 0, 1, or 2.

   (a) Find $P(X = 0)$, $P(X = 1)$, $P(X = 2)$.

   (b) Draw a picture of the probability function p where $p(k) = P(X = k)$.

   (c) Find $E[X]$.

3. A gambling books recommends the following "winning strategy" for the game of roulette. Here we will use the American wheel that has 0 and 00 on it.

It recommends that a gambler bet $1 on red. If red appears then the gambler should take their $1 profit and quit. If the gambler loses the bet, then they should play the game two more times and make additional bets of $1 on red on each of the next two spins of the roulette wheel and then quit. Let X denote the gambler's winnings or loses doing this strategy.

Recall that in Roulette a bet of red is paid 1:1. That is you get $1 for every $1 bet.

(a) Find $P(X > 0)$, that is the probability that the gambler will win some money doing this.

(b) Are you convinced that this is a "winning" strategy? Explain your answer. [Hint: Calculate $E[X]$]

4. Suppose that you play the following game. A bag has seven balls labeled $1, 2, 3, 4, 5, 6, 7$. You reach into the bag and randomly chose two balls. Order doesn't matter. The balls 1 and 7 are worth $5 each. The balls $2, 3, 4, 5, 6$ are worth $\$(-2)$ each. Let X be the amount of money won or lost playing this game.

   (a) Calculate $P(X = 10)$, $P(X = 3)$, and $P(X = -4)$

   (b) Calculate the expected value $E[X]$ of this game.

5. Two balls are chosen randomly at the same time from a bag containing 8 white balls, 4 black balls, and 2 orange balls. Suppose that we win $2 for each black ball selected and we lose $1 for each white ball selected. You don't win or lose anything for each orange ball. Let X denote the winnings or loses of this game.

   (a) Calculate the probabilities of all the possible outcomes. Calculate how much won or lost in each possible outcome.

   (b) Calculate $E[X]$.

6. Suppose from a standard 52-card deck you are given the following five cards: $4\heartsuit$, $10\heartsuit$, $Q\heartsuit$, $3\spadesuit$, $2\clubsuit$. Suppose you now discard the $3\spadesuit$ and $2\clubsuit$ from your hand (but keep the other three cards) and ask for two more cards.

(a) What is the probability that you get two more hearts so you have a flush (ie a hand with all hearts in it)?

(b) Suppose someone says: If you get a flush I'll pay you \$500. But if you don't you have to pay me \$20. Do you take the bet?

7. The following game is called Chuck-a-luck. It works as follows. You pick a number out of 1, 2, 3, 4, 5, or 6 and bet \$1 on that number. Three giant 6-sided dice are then rolled in a spinning cage. You then win \$1 for every time that your number appears on the dice. But you lose your \$1 if your number doesn't appear at all. For example, suppose that you you pick the number 1 as your number. Suppose that the dice show 1, 5, 1. Then you win \$2. If the dice showed 3, 1, 6 then you would win \$1. If the dice showed 3, 2, 6, then you would lose your \$1 bet.

   (a) Let X denote the amount of money lost or won. Let $p(i) = P(X = i)$ be the probability function for X. Calculate $p(-1)$, $p(1)$, $p(2)$, $p(3)$.

   (b) Draw a picture of p.

   (c) What is the expected value of this game?

Part 2 - You do NOT have to do problems 7 and 8 below.

They involve infinite probability spaces and won't be on any exam. Do them if interested.

---

8. Suppose that you flip a coin continually until a head occurs. Suppose that someone says: If you don't get a heads until you roll at least 3 tails then I'll pay you \$5. But if a heads occurs in the first three rolls then you must pay me \$1. Do you take the bet?

9. Consider the following experiment. Suppose we roll an 4-sided dice continually. We don't stop until a 3 is rolled.

   (a) What is a sample space S and a probability function P for such an experiment? Verify that you have a probability space.

   (b) Let A be the event that a 3 is rolled on the 3rd roll. Calculate $P(A)$.

(c) Let B be the event that a 3 is rolled within the first 3 rolls. Calculate $P(B)$.

(d) Suppose someone says this before the experiment starts: If a 3 is rolled within the first 3 rolls then I will pay you \$5, but if it doesn't then you have to pay me \$6. Do you take the bet?